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APPLICATIONS OF STATISTICS TO THE PHYSICAL SCIENCES AND BIOLOGY[1]

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1. **Random Errors.** Statistics is used basically in three ways, in the sciences. The first application deals with the study of random errors in the performance of experiments. A rather interesting example for this was the observation of the bending of light in the gravitational field of the sun in 1916 or 1917 in order to verify one of the predictions of the general theory of relativity by experiment. The corresponding experiment can only be performed during the total eclipse of the sun, which limits the possible number of observations. It appeared imperative to study the random errors of various readings as the value of the expected effect was of the order of magnitude of the standard deviation. Careful evaluation of the results was necessary to find out whether the agreement between theory and experiment was significant or could be due to chance. This was decided by the basic theory of errors.

2. **Statistical Mechanics.** A second line of application lies in the subject of classical statistical mechanics. Here we assume that the laws of forces are known but the initial position and momenta of the individual molecules are unknown. Besides, even if they were known, an individual treatment would

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not be feasible considering that we are dealing with about 10^{23} molecules and solution of the corresponding number of equations exceeds the capacity even of the best electronic computer. In this case the actaul distribution is close to the most probable one due to the large number of elements.

A rather interesting example from classical physics is the Shot Effect. This means that the electrons in a piece of wire perform a random motion and the most probable value of the electric current would be zero. Just the same careful application of statistical principles shows that irregular variations are bound to occur and consequently a small current will flow back and forth in the wire. The shot effect may be used as a subsidiary proof for the existence of an elementary quantum of electricity. This means that electric charges appear in integral multiples of a basic charge, e^{v} equal to about 1.6×10^{-19} coulomb.

A very similar effect is observed in radio tubes. The distribution of electron velocities is closely identical with the law derived from Fermi statistics, but very slight variations give rise to the noise which is inherent in the statistical character of thermionic emission. (Fermi statistics is a study of positions and momenta of particles whose spin—intrinsic angular momentum—is a half odd multiple of $\bar{h}$*.)

3. **Uncertainty Principle.** Probably the most interesting application of statistics is based on Heisenberg's principle of uncertainty. Between pairs of canonical variables there exists a law of limitation. Canonical variables are pairs of variables obeying the canonical equation of mechanics. To wit

$$\dot{p}_i = \frac{\partial H}{\partial P_i} \qquad \dot{q}_i = \frac{\partial H}{\partial q_i} \qquad p_k = \frac{\partial T}{\partial \dot{q}_k}$$

* $\bar{h} = h/\pi$, h is Planck's constant equal to about 6.6×10^{-34} Joule-second.

p_k are generalized momenta, T is the kinetic energy in arbitrary coordinates, $\dot{q}_k$ is the time derivative of q_k. p_k does not necessarily have the dimensions of momentum. H represents the Hamiltonian defined as $H = \Sigma p_i \dot{q}_i - L$. L represents the Lagrange-function equal to kinetic energy minus potential energy. For those pairs of canonically conjugate variables Heisenberg proved that any thinkable experiments will disturb the position and momentum coordinates in such a way that $\Delta q_i \; \Delta q_i \geqslant \bar{h}$.

The act of measurement interferes with the state of the system which no thinkable process can improve beyond the limit of Heisenberg's principle. The p's and the q's represent any system of values for which, at least for certain regions there is one to one correspondence to the position in space. The p's are generalized momenta. For example, if a particle is illuminated, there is a collision between the photon and the particle. If the energy of the photon is high, meaning short wavelength, the change in momentum p is appreciable; if the energy is low, meaning long wavelengths, the determination of position is inaccurate due to diffraction at the parts of the optical system used for observaion. We can make the error in p or q as small as we please, but small error in p means a big error in q and vice versa. Accurate analysis leads to the Heisenberg inequality.

Heisenberg developed a number of thought-experiments which all lead to the conclusion that no thinkable experiment can determine the position and momentum coordinates in such a manner that the product of the error is less than $\bar{h}$. With p we mean the value of the momentum coordinate after the measurement because it is the one which we need for prediction of the future. The science of quantum mechanics on which all our knowledge of atoms, molecules, nuclei and elementary particles is based consists in making probability predictions from incomplete knowledge of initial conditions. The predictions can only have a probability character. Fortunately some parameter like the total energy, if negative, can only assume discrete sets of values so that we might arrive in some cases at definite predictions of energy. The same applies to the total and to the ζ — component of angular momentum.

The movements of electrons inside an atom can not be described as in classical physics, by the orbit and the linear momentum, as only probability predictions are possible. To predict future events, Schroedinger introduced a certain function, the probability density amplitude. Its Hermitian square, multiplied by the volume element indicated the probability of finding a particle, usually an electron, or another lepton, in the volume element,

$$V - i\, h\, \psi^{*} \triangle \psi\, d\, t\ ;\ (\triangle \psi = \text{gradient}\ \psi = i \frac{\partial \psi}{\partial x} + j \frac{\partial \psi}{\partial y} + k \frac{\partial \psi}{\partial z})$$

indicates the probability of finding a particle, usually an electron in an element of momentum space. Hermitian square is the product of a function and its conjugate complex. A conjugate complex is a function which has the same real and the opposite imaginary part. So if,

$\zeta = \alpha + i\,\beta$ then the conjugate is $\zeta = \alpha - i\,\beta$.

The probability of finding an electron in a certain volume is $\iiint_{Vol} \psi\psi^{*} d\tau$ is the probability amplitude. ψ^{*} its conjugate and $d\,\tau$ is the volume element.

4. **Biological experimentation.**

Standard statistical methods, particularly those testing the null hypothesis, are widely used in Biology. The following example is very similar to applications in other branches of knowledge. One group of test animals may gain weight faster than the other group. Statistical methods can show whether the difference is due to statistical fluctuations or whether the difference is significant. It has been observed that many biological workers do not know whether a series of experiments is conclusive or not, and either publish results which are not justified, or continue working while already in full possession of significant data. In many cases empirical formulae are required. In that case a parabola of higher order is computed by the method of least squares.

Mendel's laws of heredity form another simple example of the application of statistics to Biology. During the time of the Nazi regime the government sterilized persons suffering from hereditary diseases. Assuming that four genes are needed to produce the phenotype, computation showed that it would take about a hundred thousand years to reduce hereditary diseases by about 10% as most of the carriers have just one or two defective genes.

The statistical studies of hereditary diseases gave scientific justification for the ancient law prohibiting marriage between close relatives. The probability that defective genes will match if the two spouses are unrelated is extremely small but for second degree relatives, the danger of producing offspring with hereditary defects is appreciable.